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Let $\sum_{i=1}^{n} x_i = 1$ for $x_1, \ldots, x_n > 0$. Show that the following inequality holds.

$$\left(\sum_{i=1}^{n} x_i^2\right)^2 \sqrt{n} \geq \sum_{i=1}^{n} x_i^3$$

$$\left(\sum_{i=1}^{n} x_i^2\right)^4 n \geq \left(\sum_{i=1}^{n} x_i^3\right)^2$$

$a^2 \geq b^2$

$|a| \geq |b|$

$a \geq b$

$$\left(\sum_{i=1}^{n} x_i^2\right)^3 \underbrace{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} 1\right)} \geq \left(\sum_{i=1}^{n} x_i^2\right)^3 \left(\sum_{i=1}^{n} x_i\right)^2 = \left(\sum_{i=1}^{n} x_i^2\right)^3$$

$n = 1$

$$\left(\sum_{i=1}^{n} x_i^2\right)^3 \geq \left(\sum_{i=1}^{n} x_i^3\right)^2$$

$$\left(\sum_{i=1}^{n} x_i^3\right)^2 \leq \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} x_i^4\right) \leq \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} x_i^2\right)^2 = \left(\sum_{i=1}^{n} x_i^2\right)^3$$

$\square$