Problem 1.

Suppose m and n are relatively prime positive integers such that $m > n > 0$. Moreover, m and n satisfies the following equation.

$$\frac{m^3 - n^3}{(m-n)^3} = \frac{73}{3}$$

Find $m + n$. (Source: AMC 10)

Problem 2.

Define S_n as the sum of the n^{th} powers of the roots of the polynomial $x^3 - 5x^2 + 8x - 13$. Let p, q, and r be real numbers such that $S_{n+1} = pS_n + qS_{n-1} + rS_{n-2}$ for $n \geq 2$. Find pqr. (Source: AMC 12)

Solution Let α, β, and γ be the roots of the polynomial. By definition, the following equations are true.

$$\begin{aligned} s_{k+1} &= \alpha^{k+1} + \beta^{k+1} + \gamma^{k+1} \\ s_k &= \alpha^k + \beta^k + \gamma^k \\ s_{k-1} &= \alpha^{k-1} + \beta^{k-1} + \gamma^{k-1} \\ s_{k-2} &= \alpha^{k-2} + \beta^{k-2} + \gamma^{k-2} \end{aligned}$$

The equations could be used to find the relationships between the roots and a, b, and c.

$$\begin{aligned} \alpha^{k+1} + \beta^{k+1} + \gamma^{k+1} &= a(\alpha^k + \beta^k + \gamma^k) + b(\alpha^{k-1} + \beta^{k-1} + \gamma^{k-1}) + c(\alpha^{k-2} + \beta^{k-2} + \gamma^{k-2}) \\ &= \alpha^{k-2}(c + b\alpha + a\alpha^2) + \beta^{k-2}(c + b\beta + a\beta^2) + \gamma^{k-2}(c + b\gamma + a\gamma^2) \end{aligned}$$

Continuing,

$$\alpha^{k-2}(\alpha^3 - a\alpha^2 - b\alpha - c) + \beta^{k-2}(\beta^3 - a\beta^2 - b\beta - c) + \gamma^{k-2}(\gamma^3 - a\gamma^2 - b\gamma - c) = 0$$

Notice that if $x^3 - ax^2 - bx - c = 0$, the equation will satisfy. Thus, $a = 5$, $b = -8$, $c = 13$. Therefore, $a + b + c = \boxed{\textbf{(D)} 10}$. ☐

Uploaded a new solution in AOPS!

Problem 3.

Find the coefficient of x^{28} in

$$\left(1+x+x^2+\cdots+x^{27}\right)\left(1+x+x^2+\cdots+x^{14}\right)^2.$$

(Source: AMC 12)