7.8.2025 AMC 10 Free Class Preview Homework Solution

Enoch Yu

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Problem

Find the number of real solutions of the equation $x|x| - 3|x| + 2 = 0$.

Solution First and foremost, the absolute values could be removed.

$$\begin{cases} x \geq 0 \implies x^2 - 3x + 2 = 0 \\ x < 0 \implies -x^2 + 3x + 2 = 0 \end{cases}$$

For the first case, the solutions to the equation are 1 and 2. The roots are valid since both 1 and 2 are greater than zero. The solutions to the second equation are $\frac{3\pm\sqrt{17}}{2}$. The only valid solution from the second equation is $\frac{3-\sqrt{17}}{2}$ because the root must be negative. Therefore, the number of real solutions to the equation is $\boxed{3}$. □

Problem

Find the number of pairs of non-negative integer solutions to the equation $|a - b| + ab = 1$.

Solution The absolute value could be removed.

$$\begin{cases} a \geq b \implies ab + a - b = 1 \\ a < b \implies ab - a + b = 1 \end{cases}$$

The first equation could be factored using Simon's Favorite Factoring Trick. From $(a - 1)(b + 1) = 0$, the possible ordered pairs of (a, b) are $(1, 0)$ and $(1, 1)$.

The second equation will lead to $(a + 1)(b - 1) = 0$. The possible ordered pair is $(0, 1)$. Therefore, there exists $\boxed{3}$ ordered pairs of non-negative integer solutions to the equation $|a - b| + ab = 1$. □

Problem

What is the product of all the roots of the equation $\sqrt{5|x| + 8} = \sqrt{x^2 - 16}$?

Solution Because square root functions are one to one correspondence, both sides could be squared.

$$5|x| + 8 = x^2 - 16$$

Therefore, the absolute value could be removed.

$$\begin{cases} x \geq 0 \implies x^2 - 5x - 24 = 0 \\ x < 0 \implies x^2 + 5x - 24 = 0 \end{cases}$$

The first equation gives $x = 8$. The second equation provides $x = -8$. Moreover, notice that if $x = \pm 8$, the left hand side and the right hand side will lead to a valid real equalities. Therefore, the product of all the roots of the equation is $\boxed{-64}$. □

Problem

What is the product of the real roots of the equation $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$?

Solution Let $A = x^2 + 18x + 30$. In other words, $A = 2\sqrt{A + 15}$. Therefore, $A^2 - 4A - 60 = 0$. However, A can only be 10 since A must be a positive number. $x^2 + 18x + 10 = 0$ will lead to real roots with the product of $\boxed{-10}$. $\square$

Problem

Given that $x^2 + y^2 = 10$, $\sqrt[4]{xy} + \sqrt{xy} + 27 = 29$, $x > 0$ and $y > 0$. What is $x + y$?

Solution Let $a = \sqrt[4]{xy}$. Therefore, $a^2 + a - 2 = 0$. In other words, $\sqrt[4]{xy} = -2, 1$. Because x and y are real numbers, xy must be 1. Thus $(x + y)^2 = 10 + 2$. In other words, $x + y = \boxed{2\sqrt{3}}$. $\square$