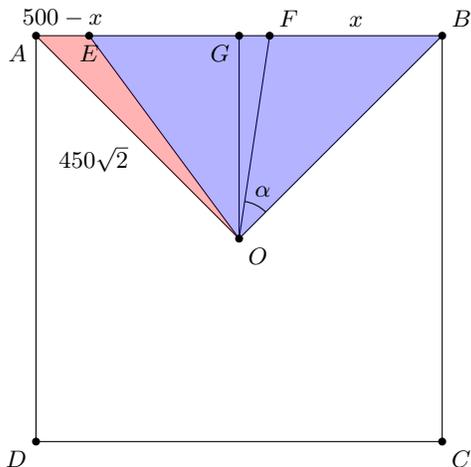


2005 AIME II Problem 12

Square $ABCD$ has center O and $AB = 900$. Points E and F are on \overline{AB} with $AE < BF$ and E between A and F such that $\angle EOF = 45^\circ$ and $EF = 400$. Find BF .

Solution



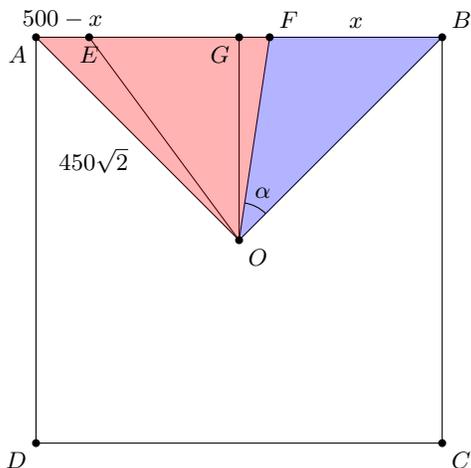
The Law of Sines could be utilized.

$$\frac{EO}{\sin 45^\circ} = \frac{450\sqrt{2}}{\sin \angle AEO} = \frac{500 - x}{\sin(45 - \alpha)}$$

$$\frac{EO}{\sin 45^\circ} = \frac{450\sqrt{2}}{\sin \angle OEB} = \frac{400 + x}{\sin(45 + \alpha)} = \frac{400 + x}{\cos(45 - \alpha)}$$

$$\frac{500 - x}{\sin(45 - \alpha)} = \frac{400 + x}{\cos(45 - \alpha)}$$

$$\frac{500 - x}{400 + x} = \tan(45 - \alpha)$$



The Law of Sines could be utilized.

$$\frac{900 - x}{\sin(90 - \alpha)} = \frac{x}{\sin(\alpha)}$$

$$\frac{x}{900 - x} = \tan \alpha$$

The addition property of tan could be used to compute for x .

$$\frac{500 - x}{400 + x} = \frac{1 - \frac{x}{900 - x}}{1 + \frac{x}{900 - x}}$$

$$\frac{500 - x}{400 + x} = \frac{900 - x - x}{900 - x + x}$$

$$900(500 - x) = (900 - 2x)(400 + x)$$

$$450000 - 900x = 360000 + 100x - 2x^2$$

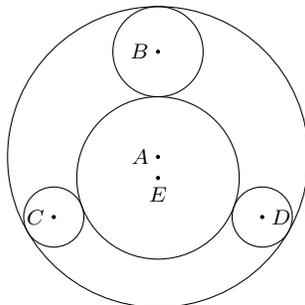
$$x^2 - 500x + 45000 = 0$$

$$\therefore x = \boxed{250 + 50\sqrt{7}}$$

□

2009 AIME II Problem 5

Equilateral triangle T is inscribed in circle A , which has radius 10. Circle B with radius 3 is internally tangent to circle A at one vertex of T . Circles C and D , both with radius 2, are internally tangent to circle A at the other two vertices of T . Circles B , C , and D are all externally tangent to circle E . Find the radius of circle E .



Solution Consider $\triangle AED$, and let the radius of circle E be r . Using the law of cosines for $\angle DAE$, the following equations could be written.

$$\begin{aligned} \cos 60^\circ &= \frac{(r-4)^2 + 8^2 - (r+2)^2}{2 \cdot (r-4) \cdot 8} \\ 8r - 32 &= -12r + 76 \\ \therefore r &= \boxed{\frac{27}{5}} \end{aligned}$$

□

Problem

The perimeter of a triangle is numerically 12 times the average of the sines of the angles of the triangle. If one side of the triangle has length 2, what are the possible measures of the angle opposite that side?

Solution Let a, b, c be the side lengths of the opposite sides of the angle A, B, C . WLOG, let $c = 2$. The following equations could be written.

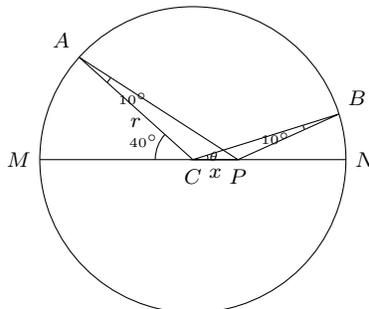
$$\begin{aligned} a + b + c &= 4(\sin A + \sin B + \sin C) \\ 2 + a + b &= 4 \left(\frac{a}{2} \sin C + \frac{b}{2} \sin C + \sin C \right) \quad \left(\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{2} \right) \\ \sin C &= \frac{1}{2} \\ \therefore C &= \boxed{30^\circ, 150^\circ} \end{aligned}$$

□

1983 AHSME Problem 30

Distinct points A and B are on a semicircle with diameter \overline{MN} and center C . The point P is on \overline{CN} and $\angle CAP = \angle CBP = 10^\circ$. If $\widehat{MA} = 40^\circ$, then what is \widehat{BN} ?

Solution



Using the law of sines, the following equation is evident.

$$\frac{\sin \angle APC}{r} = \frac{\sin 10^\circ}{x} = \frac{\sin \angle CPB}{r}$$

Because A and B are distinct points and $\sin \angle APC = \sin \angle CPB$ with $\angle APC = 30^\circ$, $\angle CPB$ must be 150° . In other words, $\theta = \boxed{20^\circ}$. □