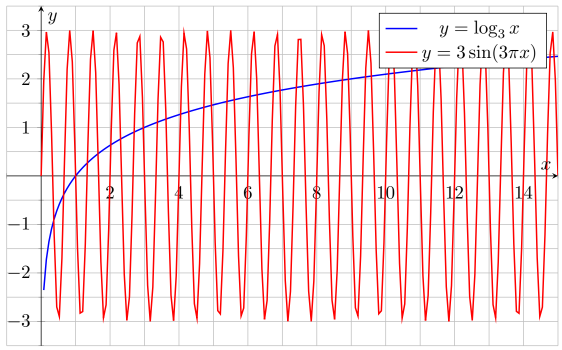
Problem

How many solutions are there to the equation $\log_3 x = 3\sin(3\pi x)$?

Solution

Key Word Graphing



Looking at the graph, it is evident that two intersection occurs for each period except for the first period, where each has one intersection. Moreover, there exists 40 periods since $\frac{27}{\frac{2}{3}} = \frac{81}{2}$.

$$\therefore 1 + 39 \cdot 2 + 2 = \boxed{81}$$

$\square$

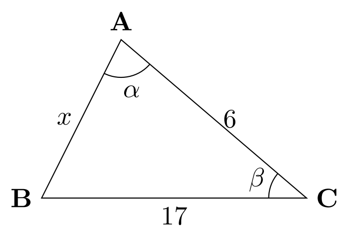
Problem

In $\triangle ABC$, $AC = 6$ and $BC = 17$. If $\tan \frac{A}{2} \tan \frac{C}{2} = \frac{2}{3}$, find AB.C

Solution

Key Word Law of Sines, Law of Cosines, Trigonometric Identities

First and foremost the situation could be drawn.



Using the law of sines, the following equation could be driven.

$$\frac{\sin\alpha}{17} = \frac{\sin\beta}{x}$$
$$\frac{\sin\beta}{\sin\alpha} = \frac{x}{17}$$

Moreover, using trigonometric identities, the following relationships could be found.

$$\begin{aligned}
\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} &= \frac{\sqrt{\frac{1-\cos\theta}{2}}}{\sqrt{\frac{1+\cos\theta}{2}}} \\
&= \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} \\
&= \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} \cdot \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} \\
&= \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} \cdot \frac{\sqrt{1-\cos\theta}}{\sqrt{1-\cos\theta}} = \frac{1-\cos\theta}{\sin\theta}
\end{aligned}$$

Therefore, the two relationships could be applied alternately.

$$\begin{aligned}
\frac{1-\cos A}{\sin A} \cdot \frac{\sin C}{1+\cos C} = \frac{1-\cos\alpha}{\sin\alpha} \cdot \frac{\sin\beta}{1+\cos\beta} &= \frac{2}{3} \\
\frac{\sin\beta}{\sin\alpha} \cdot \frac{1-\cos\alpha}{1+\cos\beta} = \frac{x}{17} \cdot \frac{1-\cos\alpha}{1+\cos\beta} &= \frac{2}{3} \\
\frac{x}{17} \cdot \frac{1 - \frac{x^2+6^2-17^2}{2\cdot 6\cdot x}}{1 - \frac{17^2+6^2-x^2}{2\cdot 17\cdot 6}} = \frac{x}{17} \cdot \frac{17}{x} \cdot \left(\frac{2\cdot 6\cdot x - x^2 - 6^2 + 17^2}{2\cdot 17\cdot 6 - 17^2 - 6^2 + x^2} \right) &= \frac{2}{3} \\
\frac{-(x-6)^2+17^2}{-(17-6)^2+x^2} = \frac{(17+x-6)((17-x+6)}{(x+11)(x-11)} &= \frac{2}{3} \\
\frac{23-x}{x-11} &= \frac{2}{3} \\
\therefore x &= \boxed{\frac{91}{5}}
\end{aligned}$$

□