

**Problem** There are four magnificent cities named *Happy*, *Content*, *Merry*, and *Jolly*. One day, countable virus named *MathVirus* has entered each city.

The number of viruses that entered the *Merry* city is the product of the number of virus that entered *Happy* city and *Content* city. Moreover, the sum of the number of virus that entered *Happy* city and *Content* city equals the number of virus that entered *Jolly* city. In addition, the number of virus that entered *Merry* city squared is equal to the product of 2025 and the number of virus that entered *Jolly* city squared.

A biologist in *Happy* city is required to count the number of the *MathVirus* that spread to *Happy* city. Please help the biologist to find all possible number of virus that dispersed to *Happy* city for further investigation! (The number of *MathVirus* for each city does not change.)

**Solution**

Let

the number of virus in *Happy* city  $H$ ,  
the number of virus in *Content* city  $C$ ,  
the number of virus in *Merry* city  $M$ ,  
the number of virus in *Jolly* city  $J$ .

$$M=HC$$

$$J=H+C$$

$$M^2=2025J^2$$

Because the number of the virus in each city is countable, the quantity of *MathVirus* in each city is natural number.

$$\therefore M = 45J$$

Because virus has entered each city,  $J$  cannot be zero.

$$\frac{M}{J} = 45$$

By substitution,

$$\frac{HC}{H+C} = 45$$

$$HC = 45H + 45C$$

$$HC - 45H - 45C = 0$$

$$HC - 45H - 45C + 2025 = 2025$$

$$(H - 45)(C - 45) = 2025$$

The possible values for  $H - 45$  are the factors of 2025.

$$H - 45 = 1$$

$$H - 45 = 3$$

$$H - 45 = 5$$

$$H - 45 = 9$$

$$H - 45 = 15$$

$$H - 45 = 25$$

$$H - 45 = 27$$

$$H - 45 = 45$$

$$H - 45 = 75$$

$$H - 45 = 81$$

$$H - 45 = 135$$

$$H - 45 = 225$$

$$H - 45 = 405$$

$$H - 45 = 675$$

$$H - 45 = 2025$$

Therefore, the possible values of  $H$  are 46, 48, 50, 54, 60, 70, 72, 90, 120, 126, 180, 270, 450, 720, 2070.

**Conclusion: The possible numbers of virus that entered to *Happy* city are 46, 48, 50, 54, 60, 70, 72, 90, 120, 126, 180, 270, 450, 720, 2070.**