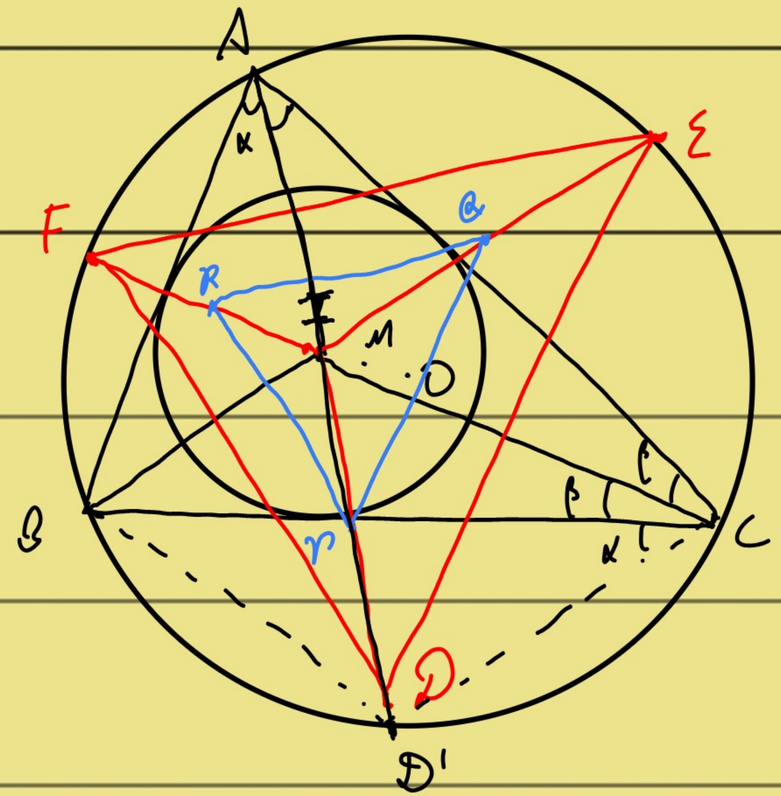
Enoch Yu

2009 KMO I

Given a triangle ABC with incenter I and circumcenter O, let the circumcenters of triangles BIC, CIA, AIB be D, E, F respectively. If P, Q, R are the midpoints of DI, EI, FI respectively and M is the circumcenter of $\triangle PQR$, show that M is the midpoint of IO.



$\triangle DEF, \triangle PQR$

$D, E, F \in \omega$?

$\hookrightarrow$ I is the homothetic center,

$\therefore$ I, M, O are collinear, $IM:MO$.

$\angle D'IC = \alpha+\beta = \angle ICD'$

$\angle D'BI = \angle D'IB$

$\Rightarrow D'B = D'I = D'C \quad \therefore D' \equiv D$

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