Enoch Yu

2010 KMO 1.

Show that $7^{2^{20}} + 7^{2^{19}} + 1$ is divisible by at least 21 distinct prime numbers.

21 20 19

$\eta^{2^n} + \eta^{2^{n-1}} + 1 \Rightarrow n+1$ → coprime?

$(\eta^{2^n} + \eta^{2^{n-1}} + 1)(\eta^{2^n} - \eta^{2^{n-1}} + 1)$

$= \eta^{2^{n+1}} - \eta^{2^n}\eta^{2^{n-1}} + \eta^{2^n} + \eta^{2^{n-1}}\eta^{2^n} - \eta^{2^n} + \eta^{2^{n-1}} + \eta^{2^n} - \eta^{2^{n-1}} + 1$

$= \eta^{2^{n+1}} + \eta^{2^n} + 1$

3·19

$\eta^{2^{20}} + \eta^{2^{19}} + 1 = (\eta^{2^{19}} - \eta^{2^{18}} + 1)(\eta^{2^{18}} - \eta^{2^{17}} + 1) \cdots (\eta^2 - \eta + 1)(\eta^2 + \eta + 1)$

(a terms; 20 terms)

$(\eta^{2^n} + \eta^{2^{n-1}} + 1, \eta^{2^n} - \eta^{2^{n-1}} + 1)$

$= (\eta^{2^n} + \eta^{2^{n-1}} + 1, \eta^{2^n}) = (\eta^{2^{n-1}} + 1, \eta^{2^n}) = 1$

□