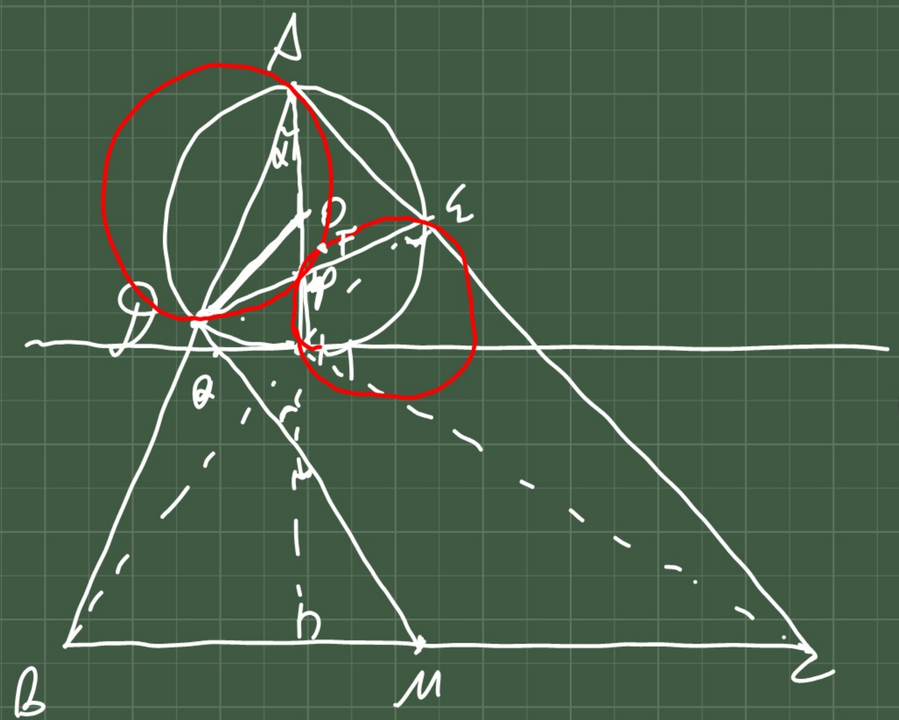
2020 IMO Problem 2

Enoch Yu

Given an acute triangle ABC, let H and M be the orthocenter and the midpoint of BC respectively. Let D and E be the intersection between the circle with diameter AH and sides AB and AC respectively. Let P be the intersection of AH and DE and let Q be the intersection of DM and the line perpendicular to AH that passes through H. Show that P, Q, B are collinear.



Solution 1.

$OD \perp DM$?

$\iff$

$\angle DOH = \angle DMB$

$\alpha = \angle DAH = \angle DEH = \angle PCB$

$\therefore \angle DMB = 2\alpha = \angle DOH$

$AB, PF, EM \Rightarrow B$

$B, P, F \in \ell$

$(ODQH) \Rightarrow OQ$ diameter

$\angle DFH = \angle DFP + \angle PFH = \alpha + \alpha = 2\alpha = \angle DOH$

$\therefore (ODQHF)$

$\square$

$OF \perp FP$?

$\angle OFP = \angle OFD + \angle DFP$

$= \angle OHD + \angle DAP$

$= 90^\circ$

Solution 2.

$P = DE \cap AH$

$Q = DD \cap HH$

$B = AD \cap HE$

HH	A D	DE
DD	EH	HA

$(HHADDE)$

$\square$